Discussion on the Application of Counterexamples in the Teaching of Real Variable Functions

Yong Tie

Qujing Normal University, Yunnan, Qujing, 655011, China

Keywords: Real Variable Function Course; Teaching; Counterexample Application

Abstract: As the name implies, the real variable function is a function with real numbers as independent variables. In the process of solving problems in the real problem, using forward reasoning, the use of forward reasoning often cannot solve more complicated mathematical questions. Therefore, timely adoption is reversed. The applied method not only provides a thorough understanding of mathematical theory and concepts, but also provides a variety of problem-solving ideas for solving similar types of questions.

1. Introduction

The counterexample solving method is often widely used in logic. Assume that a false proposition is listed. To make this proposition stand, an example is given as a solution entry point. This example is called a counterexample. This method solves related mathematical and philosophical problems, which is the concept of anti-example application. In the course of real variable function teaching, if you can correctly use the method of counterexample application, it will not only simplify some complicated problems, but also receive the ideal teaching effect.

2. The Counterexample Application Opens a New Era of Solving The Real Variable Function

For most students, the real variable function course is theoretically strong and professional. When solving the mathematical problem of the real variable function, it is often impossible to start. In solving problems, students often solve problems in a positive way based on the methods taught by the teacher, but they do not receive the desired results. To this end, in the teaching classroom, the teacher should use the method of counterexample application to start teaching according to the characteristics of the question type [1].

For the simple complete set concept, if students follow the correct reasoning, they will enter the cognitive misunderstanding of the complete set. It is easy to misunderstand that the intersection and union of the complete set is a complete set, in order to correct the students in time. Our mistakes, at this time, can introduce counterexamples, so that students have a correct understanding of the concept of complete sets.

For example, the set P1=[1, 2], P2=[2, 3] is a complete set, but the intersection P1∩P2={2}, obviously, this intersection is not complete. The set [1, 3/2], [3/2, 7/4], ..., [2-1/2n-1, 2-1/2n] are all complete sets, but their union is not a complete set. Through this counterexample application, the concept of the complete set can be well explained. At the same time, the students are clear about the problem solving process and will not generate any objection. Therefore, the counterexample can be applied to any problem type of the real variable function. The results are also obvious.

The starting point of classroom education is to teach students how to learn the alternative methods when solving problems, and the counterexample application is just one of the most direct and effective methods to solve the various problems of the real variable function.

3. The Practical Significance of Taking Counterexamples

In recent years, the use of counterexamples to solve more complex mathematical questions has become a method of practical application of many mathematics teachers, and received the ideal

  DOI: 10.25236/iwass.2018.194

teaching effect in the actual problem solving process. For students, the method of counterexample application has practical guiding significance, which is beneficial to the development of students' divergent thinking, innovative thinking, and correct and rigorous learning attitude.

Some students have some vagueness about the concept of real variable function, and it is often easy to confuse the basic definitions of two different types of questions. The use of counterexamples to solve problems can eliminate students' fuzzy cognitive thoughts and produce clear solutions to relevant problems. The application of counterexamples can grasp the essence of mathematical concepts and solve some complex problems of real variable functions. [2].

For the concept of collections in real-time functions, the concept itself is very abstract. Many students often face the esoteric and incomprehensible collection definitions when they first contact the collection, and they are overwhelmed. For example, the concept of a true subset, if the set $A \subseteq B$, but the existence of the element $X \in B$, and the element X does not belong to the set A, we call the set A is the true subset of the set B. At first glance, the concept of true subsets is not a direct narrative, but an interpretation of some unknown elements, which makes many students discouraged. If the teacher takes a counterexample, it can be easily solved. It can be seen from the definition that a finite set and a true sub-set cannot have the same cardinality, and it is concluded that all sets and true subsets cannot have the same cardinality. In this case, a counterexample is set, the integer set is associated with it. The natural subset of the true subset has the same cardinality. Through this counterexample, students can intuitively understand the concept of the true subset and lay the foundation for the next step of learning. At the same time, students have the difference between the finite number set and the infinite number limit. It can also be easily distinguished.

It is possible to intuitively understand mathematical propositions and related theorems and laws. In the teaching process of real variable function, the appropriate introduction of counterexample application can make beginners understand some mathematical concepts, conclusions, and laws more intuitively. Especially for some mathematical propositions, counterexamples are used to rebut the wrong propositions. Quickly get the true proposition, so the beginning of the teaching of the real variable function should lead to the concept of application of the counter-example, so that students can keep up with the progress of learning as soon as possible. The counterexample application, while making a correct understanding of the basic theorem and proposition, can also effectively promote students to form a more rigorous reasoning thinking and a good habit of attaching importance to problem solving conditions [3].

For example, for measure theory, the actual definition is to specify a number for some subset of a given set, which can be compared to size, volume, probability, and so on. The measure theory has a very important position in mathematical problem analysis and probability theory. It is a branch of real analysis. The research object has σ algebra, measure, measurable function and integral, and its importance is reflected in probability theory and statistics. If a teacher adopts a teaching method that is counter-examplely applied, it will be of great help to students who have just come into contact with the measurement theory.

Mathematics has two important problem-solving modes. It enumerates both counterexamples and actual proofs. The real-time functions belong to the category of higher mathematics. Some students are weak in mathematics in the middle and high school, and they do not understand the basic concepts and definitions. Thorough the practice of the problem is relatively small, so when you touch the real function, there will be fear. The learning real variable function aims to improve students' creative ability, innovation consciousness and logical thinking ability. If the foundation is not strong, it is difficult to learn the real variable function, and then lose the interest in learning mathematics.

At this time, the teacher should correctly guide the students, the correct attitude towards learning mathematics, take the students as the main body in the classroom teaching, start from the students' interest points, and introduce the teaching of counterexamples when talking about the real variable function theorem that is difficult to understand. The method eliminates students' rebellious emotions and stimulates students to learn mathematics.

In a certain learning stage of student learning, teachers are influenced by the traditional teaching methods, and often habitually enter the sea of questions, face a large number of mathematical questions, bury their heads to solve problems, and some correct and reasonable problem solving methods are thrown Behind the brain, over time, students develop a bad habit and become a vicious circle, which is extremely unfavorable for learning mathematics. Therefore, teachers should change the students' existing thinking and grasp the opportunity to use the counter-example solving skills to introduce students into the correct problem-solving track.

In the process of learning the real variable function, the students often encounter some typical problems that are more complicated and have divergent thinking. At this time, the teacher can introduce counterexamples in the teaching class, and at the same time create relevant problem situations according to the question type itself, so that the students In the process of solving problems, the immersive feeling is generated, and it is not affected by external factors, and the whole mind is devoted to the process of solving the problem. Before solving the problem, teachers can encourage students to be good at finding problems, solving extended problems, paying attention to cultivating students' divergent thinking, and effectively exerting their own creativity and innovation. This has a positive reality for solving problems.

The counterexample application is to construct a wrong theoretical proposition, so that the original proposition can be correctly recognized through the counterexample application. The counterexample has a strong intuitiveness and persuasiveness in the analysis of the error. Appropriate use of the counterexample teaching can promptly find the error. For example, some students think that the continuous function must be a bounded variable function. Teachers can eliminate this misconception in the bud by enumerating the counterexamples.

The various theorems and conclusions involved in the real variable function are established by the proof method of positive thinking. However, some arguments do not seem to hold true. At this time, it is necessary to apply the counterexamples to make some seemingly unsatisfactory theorems and conclusions accurately confirmed. For example, the following question, for the sum of several semi-continuous functions, may be a function with no semi-continuous function, solved by the application of the counterexample, and the answer is: the function f (x) , g (x) , h (x) are each semi-continuous, but the function of their sum is a function with no semi-continuous. Using the normal problem-solving thinking mode, it is impossible to draw correct conclusions immediately. Therefore, the counterexample application can greatly improve the efficiency of solving problems, so that students can quickly accept the relevant knowledge of real variables.

Counterexample application can enable students to quickly understand the scope of application of the definition. Any mathematical problem has three conditions, including: sufficient conditions, necessary conditions, and sufficient and necessary conditions. The real function is no exception. However, in the actual teaching process, many students will not pay attention to the conditions and conclusions of the questions. The relationship between the two links makes the use of some wrong theorems and conclusions in solving problems. The problem of solving the problem is wrong, and the conclusion of the problem will be biased. Therefore, it should be combined with the counterexample to explain the theorem. [4].

For example, the conclusion of the Yegorov's theorem can not be strengthened to eliminate the set of zeros. The {fn} uniformly converges to f. Take E=[0, 1] to define the function sequence. The function column fn of this problem is a continuous function on the closed interval [0, 1], so the measurable function on the closed interval [0, 1] also satisfies the condition of the Eropo B theorem, but there is no measure The set of zeros e $\cup$ E, such that {fn} uniformly converges to zero on E. Through this, it is fully explained that when using the Eropo B theorem, the student cannot strengthen the conclusion to the point where the measure with zero is removed. {fn} uniformly converges to f. In addition, in the rational application of the Yegorov theorem and its inverse theorem, Lujin theorem and other conclusions, the method of counterexample application can also be used to verify that the conditions and conclusions of the conditional theorem cannot be arbitrarily strengthened and weakened. Through this method, the classroom teaching effect can be improved, and the mathematics theorem and its scope of application can be clearly distinguished by the students.

4. The Practical Application of Counterexamples

The application effect of the counterexample in the real variable function has been widely recognized by the teachers. The counterexample application provides a convenient solution method for the students. The following is an example of the measurable function and the continuous function to verify the effect of the counterexample application.

The continuous function and the measurable function are defined as: defined in the interval (a, b), if the left limit is continuous at the point a. A function is continuous at each point in the open interval (a, b), then , b) continuous, if it is continuous at point a, and point b is left continuous, then it is continuous in closed interval [a, b], if it is continuous in the whole domain, it is called continuous function. If a function is at a certain point in the domain If the left and right are continuous, the function is said to be continuous at this point, otherwise it is not continuous at this point [5].

Let f be a real function defined on the measurable set E. If for each real number, the set E[f>a] is always measurable (Lebes measurable), then f is the (Lebesberg) measurable function theorem defined on E. Let f be defined in the measurable set The real function on E, any of the following conditions are on E (Leberg). Let (X, F) be a measurable space and E be a measurable set. f: $E \rightarrow R^*$ is a function defined on E. If there is always $\{x \in E: f(x)<a\} \in F$ for any real number a, then f is called F-measurable function on E (referred to as measurable function on E). If the measurable space is taken as the Lebesgue measurable space on R^n. E is the Lebesgue measurable set in R^n. Then the measurable function on E becomes the Lebesgue measurable function. If the measurable space is taken as the Borel measurable space on R^n, and E is the Borel set in R^n, the measurable function on E is called the Borel measurable function.

As can be seen from the definition, the measurable function is a "continuous" function (ie, removing a point set that can be arbitrarily small). The conclusion of Lujin's theorem cannot be changed to: there is a closed subset, so that f (x) is a continuous function on F, and m (EF) =0. And by the definition of f (x), it is at [0, 1] can be measured; but for any zero measure set, f (x) is not continuous on [0, 1]\M. Therefore, the set F satisfying the condition does not exist. It can be seen that the counterexample application of the problem solving method makes some complicated theories and questions simple and clear.

5. Conclusion

The position of the counterexample application in the solution of the real variable function is irreplaceable. In the teaching of the real variable function, if the teacher correctly and rationally uses the counterexample teaching, the students can learn mathematics, understand the abstract mathematical concepts and cultivate students' learning.

References

[1] Shen Chunfang, Yang Liu. On the counterexamples in the course of "real variable function" [J]. Science and Education Wenhui, 2017(9): 50-51.

[2] Zou Qingyun, Zhou Qiyuan, Mei Hanfei. Discussion on the Teaching of Real Variable Function Courses in Local Universities [J].Journal of Science of Science and Technology, 2017, 37(7):75-77.

[3] Feng Shuxia. The Application of Counterexamples in the Teaching of Real Variable Functions [J]. Advanced Mathematics Research, 2015, 18(1): 112-114

[4] Li Kun. Reform and Practice of Real Variable Function Teaching Based on the Concept of Innovative Education [J].Journal of Science of Teachers' College, 2018, 38(4):54-56, 59.

[5] Xu Gang. Application of Reverse Thinking in the Teaching of Real Variable Functions [J]. Weekly, 2017, 35(35):26-27.